## Trigonometric function

## 1

In the coordinate plane with point O as the origin, take the positive part of the $x$-axis as the starting line and illustrate the radius of motion OP rotated by the following angle. Also, express the general angle $\theta$ represented by the radius of motion OP in the form $\theta=\alpha+360^{\circ} \times n\left(0^{\circ} \leqq \alpha<360^{\circ}, n\right.$ is an integer $)$, and answer in what quadrant the angle is.
(1) $800^{\circ}$
(2) $-200^{\circ}$



Math-Aquarium【Exercises] Trigonometric function

2 Rewrite the following angles in degrees to arc degrees and arc degrees to degrees, respectively.
(1) $135^{\circ}$
(2) $-108^{\circ}$
(3) $\frac{\pi}{2}$
(4) $-\frac{13}{10} \pi$

Math-Aquarium【Exercises] Trigonometric function

3
Find the arc length $l$ and area $S$ of a fan shape whose radius is 9 and whose central angle is $\frac{2}{3} \pi$.

Math-Aquarium【Exercises] Trigonometric function

4 Find the values of $\sin \theta, \cos \theta$, and $\tan \theta$, respectively, when $\theta$ has the following values.
(1) $\frac{5}{3} \pi$
(2) $-\frac{3}{4} \pi$

Math-Aquarium【Exercises] Trigonometric function

5
If $\theta$ is an angle in the fourth quadrant and $\cos \theta=\frac{1}{3}$, find the values of $\sin \theta$ and $\tan \theta$, respectively.

Math-Aquarium【Exercises] Trigonometric function

6
When $\sin \theta+\cos \theta=\frac{1}{2}$, find the value of the following expression.
(1) $\sin \theta \cos \theta$
(2) $\sin ^{3} \theta+\cos ^{3} \theta$

Math-Aquarium【Exercises] Trigonometric function

7 Find the following values.
(1) $\sin \frac{100}{3} \pi$
(2) $\tan \left(-\frac{3}{4} \pi\right)$
(3) $\sin \frac{3}{10} \pi+\cos \frac{4}{5} \pi$

8
(1) Graph the following functions. Find its period.
(1) $y=-\frac{1}{2} \cos \theta$
(2) $y=\tan 2 \theta$
(3) $y=\sin \left(\theta+\frac{\pi}{2}\right)+1$
(2) For the functions (1) through (3) in (1), answer which are even functions and which are odd functions, respectively.

Math-Aquarium【Exercises] Trigonometric function

9 Solve the following equations and inequalities for $0 \leqq \theta<2 \pi$.
(1) $\sin \theta=-\frac{1}{\sqrt{2}}$
(2) $\quad \cos \theta>\frac{1}{2}$

Math-Aquarium【Exercises] Trigonometric function

## 10

(1) Solve the equation $2 \sin \left(\theta-\frac{\pi}{6}\right)=-\sqrt{3} \quad$ for $0 \leqq \theta<2 \pi$.
(2) Solve the following equations and inequalities for $0 \leqq \theta<2 \pi$.
(1) $2 \sin ^{2} \theta+3 \cos \theta-3=0$
(2) $2 \sin ^{2} \theta+3 \cos \theta-3 \geqq 0$

Math-Aquarium【Exercises] Trigonometric function

## 11

Find the maximum and minimum values of the function $y=\sin ^{2} \theta+\cos \theta$ when $0 \leqq \theta<2 \pi$.
Also, find the value of $\theta$ at that time.

Math-Aquarium【Exercises] Trigonometric function

12 Find the following values.
(1) $\sin 15^{\circ}$
(2) $\cos 195^{\circ}$
(3) $\tan \frac{5}{12} \pi$

Math-Aquarium【Exercises] Trigonometric function

## 13

$0<\alpha<\frac{\pi}{2}, \quad \pi<\beta<\frac{3}{2} \pi$ and $\cos \alpha=\frac{12}{13}, \quad \sin \beta=-\frac{3}{5}$, find the following values.
(1) $\sin (\alpha-\beta)$
(2) $\cos (\alpha-\beta)$

Math-Aquarium【Exercises] Trigonometric function

14 Find the acute angle $\theta$ formed by the two lines $y=5 x$ and $2 x=3 y$.

Math-Aquarium【Exercises] Trigonometric function

## 15

Find the values of $\sin 2 \alpha, \cos 2 \alpha$, and $\tan 2 \alpha$ when $\frac{\pi}{2}<\alpha<\pi$ and $\sin \alpha=\frac{1}{4}$.

Math-Aquarium【Exercises] Trigonometric function

## 16

Find the values of $\sin \frac{\alpha}{2}, \cos \frac{\alpha}{2}$, and $\tan \frac{\alpha}{2}$ when $\frac{3}{2} \pi<\alpha<2 \pi$ and $\sin \alpha=-\frac{4}{5}$.

Math-Aquarium【Exercises] Trigonometric function

17 Solve the following equations and inequalities for $0 \leqq \theta<2 \pi$.
(1) $\sin 2 \theta=-\sqrt{2} \cos \theta$
(2) $\cos 2 \theta<3 \cos \theta+1$

Math-Aquarium【Exercises] Trigonometric function

18 Transform the following equation into the form $r \sin (\theta+\alpha)$. However, $r>0$ and $-\pi<\alpha \leqq \pi$.
(1) $-\sin \theta+\cos \theta$
(2) $\sqrt{3} \sin \theta-3 \cos \theta$

Math-Aquarium【Exercises] Trigonometric function

19 Solve the following equations and inequalities for $0 \leqq \theta<2 \pi$.
(1) $\sin \theta-\sqrt{3} \cos \theta-1=0$
(2) $\sqrt{2} \sin \theta+\sqrt{2} \cos \theta \leqq-\sqrt{3}$

Math-Aquarium【Exercises] Trigonometric function

## 20

Find the maximum and minimum values of the function $y=\sqrt{3} \sin \theta+\cos \theta-1$ when $0 \leqq \theta<2 \pi$. Also, find the value of $\theta$ at that time.

Math-Aquarium【Exercises] Trigonometric function

## Study 1

If the equation $\sin ^{2} \theta+\cos \theta-a=0$ has three solutions with $0 \leqq \theta<2 \pi$, find the value of the constant $a$.

Math-Aquarium【Exercises] Trigonometric function

## Study 2 Find the following values.

(1) $\sin 105^{\circ} \cos 15^{\circ}$
(2) $\cos 15^{\circ} \cos 75^{\circ}$
(3) $\sin 15^{\circ}+\sin 75^{\circ}$
(4) $\cos 15^{\circ}-\cos 105^{\circ}$

## Study 3 When $0 \leqq \theta<2 \pi$, answer the following questions.

(1) Find the maximum and minimum values of the function $y=\sin \theta \cos \theta-\sqrt{3} \sin ^{2} \theta$.

Also, find the value of $\theta$ at that time.
(2) Find the maximum and minimum values of the function $y=\sin 2 \theta-2 \sin \theta+2 \cos \theta$.

Also, find the value of $\theta$ at that time.

