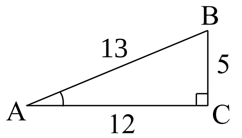
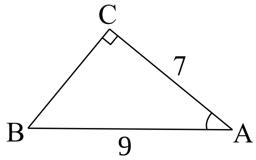
Graphics and Measurement

1

(1) Find the values of sin$A$, cos$A$, and tan$A$ in right triangle ABC shown on the right.



(2) Find the values of sin$A$, cos$A$, and tan$A$ in right triangle ABC shown on the right.

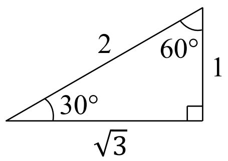


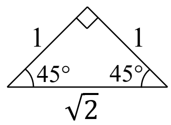
(3) Using the right triangle shown on the right as a reference, find the values of the following trigonometric ratios.

① sin45°

② cos60°

③ tan30°



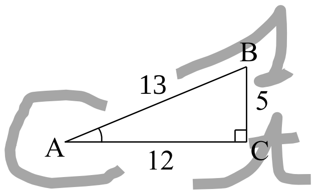


solution

(1) $\sin A = \dfrac{\text{BC}}{\text{AB}} = \mathbf{\dfrac{5}{13}}$,

$\cos A = \dfrac{\text{AC}}{\text{AB}} = \mathbf{\dfrac{12}{13}}$,

$\tan A = \dfrac{\text{BC}}{\text{AC}} = \mathbf{\dfrac{5}{12}}$



(2) Change the orientation of right triangle ABC as shown on the right.

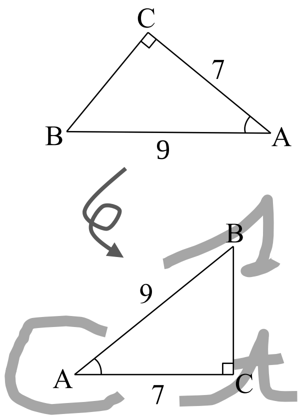
By the Pythagorean theorem, we have BC^2＝9^2－7^2＝32 .

Since BC＞0, it is BC＝$4\sqrt{2}$.

Therefore, $\sin A = \mathbf{\dfrac{4\sqrt{2}}{9}}$,

$\cos A = \mathbf{\dfrac{7}{9}}$,

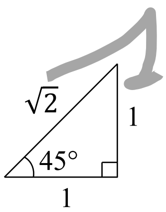
$\tan A = \mathbf{\dfrac{4\sqrt{2}}{7}}$.

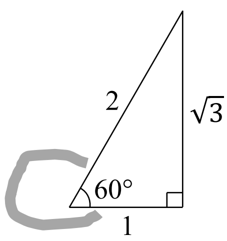


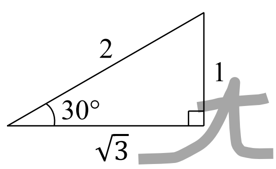
(3) ① $\sin 45° = \mathbf{\dfrac{1}{\sqrt{2}}}$

② $\cos 60° = \mathbf{\dfrac{1}{2}}$

③ $\tan 30° = \mathbf{\dfrac{1}{\sqrt{3}}}$



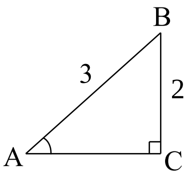




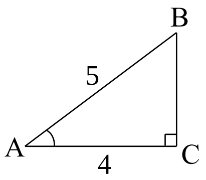
2

Using the table of trigonometric ratios, find the approximate size A of angle A in right triangle ABC in the following figures.

(1)



(2)



(3)

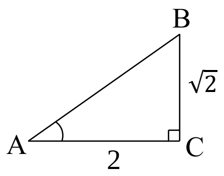


Table of trigonometric ratios

A	sin$A$	cos$A$	tan$A$
~			
25°	0.4226	0.9063	0.4663
26°	0.4384	0.8988	0.4877
27°	0.4540	0.8910	0.5095
28°	0.4695	0.8829	0.5317
29°	0.4848	0.8746	0.5543
30°	0.5000	0.8660	0.5774
31°	0.5150	0.8572	0.6009
32°	0.5299	0.8480	0.6249
33°	0.5446	0.8387	0.6494
34°	0.5592	0.8290	0.6745
35°	0.5736	0.8192	0.7002
36°	0.5878	0.8090	0.7265
37°	0.6018	0.7986	0.7536
38°	0.6157	0.7880	0.7813
39°	0.6293	0.7771	0.8098
40°	0.6428	0.7660	0.8391
41°	0.6561	0.7547	0.8693
42°	0.6691	0.7431	0.9004
43°	0.6820	0.7314	0.9325
44°	0.6947	0.7193	0.9657
45°	0.7071	0.7071	1.0000
~			

The table above is an excerpt of the relevant parts of this file.

solution

(1) $\sin A = \frac{2}{3} \approx 0.6667$.

From the table of trigonometric ratios, sin41° = 0.6561, sin42° = 0.6691.

The value of sin42° is closest to 0.6667, so **$A \approx 42°$**.

(2) $\cos A = \frac{4}{5} = 0.8$.

From the table of trigonometric ratios, cos36° = 0.8090 and cos37° = 0.7986.

The value of cos37° is closest to 0.8, so **$A \approx 37°$**.

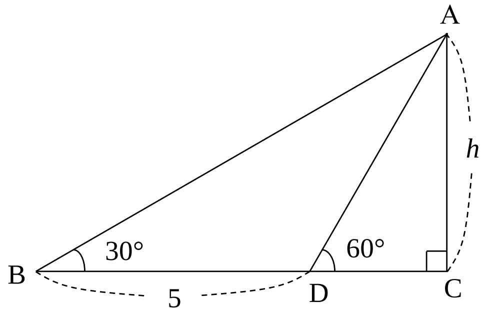
(3) $\tan A = \frac{\sqrt{2}}{2}$. $\sqrt{2} = 1.4142\cdots\cdots$, so it is $\frac{\sqrt{2}}{2} \approx 0.7071$.

From the table of trigonometric ratios, tan35° = 0.7002 and tan36° = 0.7265.

The value of tan35° is closest to 0.7071, so **$A \approx 35°$**.

3

Find h in the figure on the right.



solution

In $\triangle$ ADC, it is DC : AC $= 1 : \sqrt{3}$, therefore DC : $h = 1 : \sqrt{3}$. Thus, $\mathrm{DC} = \dfrac{h}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}h$.

In $\triangle$ ABC, it is AC : BC $= 1 : \sqrt{3}$, therefore $h : \left(5 + \dfrac{\sqrt{3}}{3}h\right) = 1 : \sqrt{3}$.

Thus, $5 + \dfrac{\sqrt{3}}{3}h = \sqrt{3}h$. From $\dfrac{2\sqrt{3}}{3}h = 5$, $\boldsymbol{h} = \dfrac{15}{2\sqrt{3}} = \boldsymbol{\dfrac{5\sqrt{3}}{2}}$.

Alternative solution

Using the fact that △ABD is an isosceles triangle with AD＝DB＝5,
it can also be obtained from AD : AC＝2 : $\sqrt{3}$.

4

θ shall be an acute angle.

(1) Find the values of $\sin\theta$ and $\tan\theta$ when $\cos\theta = \frac{1}{3}$.

(2) Find the values of $\sin\theta$ and $\cos\theta$ when $\tan\theta = \frac{1}{7}$.

solution

(1) From $\sin^2\theta + \cos^2\theta = 1$, $\sin^2\theta = 1 - \cos^2\theta$. Substituting $\cos\theta = \frac{1}{3}$ for this, we have

$\sin^2\theta = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$. Since θ is an acute angle, $\sin\theta > 0$.

Therefore, it becomes $\boldsymbol{\sin\theta} = \sqrt{\frac{8}{9}} = \boldsymbol{\frac{2\sqrt{2}}{3}}$. Also, it becomes $\boldsymbol{\tan\theta} = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}}{3} \div \frac{1}{3} = \frac{2\sqrt{2}}{3} \times 3 = \boldsymbol{2\sqrt{2}}$.

(2) Substituting $\tan\theta = \frac{1}{7}$ for $1 + \tan^2\theta = \frac{1}{\cos^2\theta}$, we have $\frac{1}{\cos^2\theta} = 1 + \left(\frac{1}{7}\right)^2 = 1 + \frac{1}{49} = \frac{50}{49}$.

Therefore, $\cos^2\theta = \frac{49}{50}$. Since θ is an acute angle, $\cos\theta > 0$. Thus, $\boldsymbol{\cos\theta} = \sqrt{\frac{49}{50}} = \boldsymbol{\frac{7}{5\sqrt{2}}}$.

Also, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to $\boldsymbol{\sin\theta} = \tan\theta \cdot \cos\theta = \frac{1}{7} \cdot \frac{7}{5\sqrt{2}} = \boldsymbol{\frac{1}{5\sqrt{2}}}$.

Alternative solution

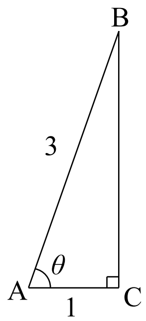
(1) From $\cos\theta = \frac{1}{3}$, draw right triangle ABC

with AB＝3, AC＝1, and ∠C＝90°.

By the Pythagorean theorem, it is $BC^2 = AB^2 - AC^2 = 3^2 - 1^2 = 8$.

$BC = \sqrt{8} = 2\sqrt{2}$ From $BC > 0$.

Therefore, $\sin\theta = \frac{2\sqrt{2}}{3}$, $\tan\theta = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$.



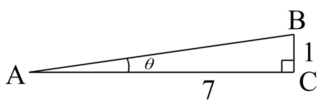
(2) From $\tan\theta = \frac{1}{7}$, draw right triangle ABC

with AC＝7, BC＝1, ∠C＝90°.

By the Pythagorean theorem, it is $AB^2 = AC^2 + BC^2 = 7^2 + 1^2 = 50$.

$AB = \sqrt{50} = 5\sqrt{2}$ From $AB > 0$.

Therefore, $\sin\theta = \frac{1}{5\sqrt{2}}$, $\cos\theta = \frac{7}{5\sqrt{2}}$.



5

Express the following trigonometric ratios for angles smaller than 45°.

(1) $\sin 80^\circ$ (2) $\cos 50^\circ$ (3) $\tan 64^\circ$

solution

(1) $80^\circ = 90^\circ - 10^\circ$, and $\sin(90^\circ - \theta) = \cos\theta$, so

$$\sin 80^\circ = \sin(90^\circ - 10^\circ) = \mathbf{\cos 10^\circ}.$$

(2) $50^\circ = 90^\circ - 40^\circ$, and $\cos(90^\circ - \theta) = \sin\theta$, so

$$\cos 50^\circ = \cos(90^\circ - 40^\circ) = \mathbf{\sin 40^\circ}.$$

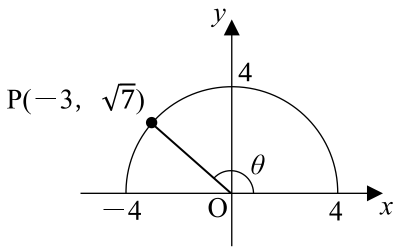
(3) $64^\circ = 90^\circ - 26^\circ$, and $\tan(90^\circ - \theta) = \dfrac{1}{\tan\theta}$, so

$$\tan 64^\circ = \tan(90^\circ - 26^\circ) = \mathbf{\frac{1}{\tan 26^\circ}}.$$

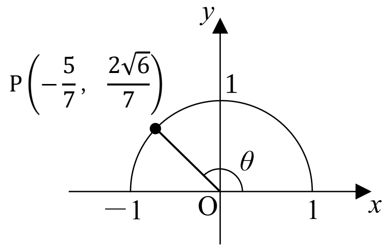
6

(1) In the following figures, find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

①



②



(2) Find the values of the following trigonometric ratios.

① $\sin 120°$　　② $\cos 135°$　　③ $\tan 150°$

solution

(1) ① Since $r=4$ and the coordinates of point P are $(-3,\ \sqrt{7})$,

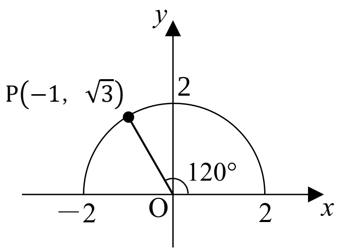
$$\sin\theta=\frac{\sqrt{7}}{4},\qquad \cos\theta=\frac{-3}{4}=-\frac{3}{4},\qquad \tan\theta=\frac{\sqrt{7}}{-3}=-\frac{\sqrt{7}}{3}.$$

② Since $r=1$ and the coordinates of point P are $\left(-\frac{5}{7},\ \frac{2\sqrt{6}}{7}\right)$,

$$\sin\theta=\frac{\frac{2\sqrt{6}}{7}}{1}=\frac{2\sqrt{6}}{7},\qquad \cos\theta=\frac{-\frac{5}{7}}{1}=-\frac{5}{7},\qquad \tan\theta=\frac{\frac{2\sqrt{6}}{7}}{-\frac{5}{7}}=-\frac{2\sqrt{6}}{5}.$$

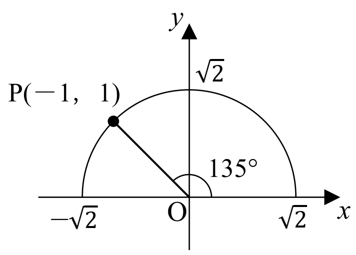
(2) ① When $\theta=120°$, point P can be taken as shown in the figure on the right.

Therefore, $\sin 120°=\dfrac{\sqrt{3}}{2}$.



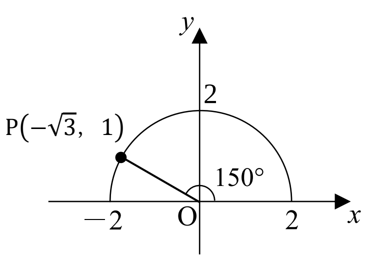
② When $\theta=135°$, point P can be taken as shown in the figure on the right.

Therefore, $\cos 135°=\dfrac{-1}{\sqrt{2}}=-\dfrac{1}{\sqrt{2}}$.



③ When $\theta=150°$, point P can be taken as shown in the figure on the right.

Therefore, $\tan 150°=\dfrac{1}{-\sqrt{3}}=-\dfrac{1}{\sqrt{3}}$.



7

Express the following trigonometric ratios for angles smaller than 90°.

(1) $\sin 160°$　　(2) $\cos 105°$　　(3) $\tan 128°$

solution

(1) $160° = 180° - 20°$, and $\sin(180° - \theta) = \sin\theta$, so

$\sin 160° = \sin(180° - 20°) = $ **$\sin 20°$** .

(2) $105° = 180° - 75°$, and $\cos(180° - \theta) = -\cos\theta$, so

$\cos 105° = \cos(180° - 75°) = $ **$-\cos 75°$** .

(3) $128° = 180° - 52°$, and $\tan(180° - \theta) = -\tan\theta$, so

$\tan 128° = \tan(180° - 52°) = $ **$-\tan 52°$** .

8

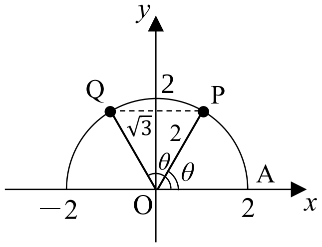
When $0° \leqq \theta \leqq 180°$, find θ satisfying the following equations.

(1) $\sin\theta = \frac{\sqrt{3}}{2}$ (2) $\cos\theta = -\frac{1}{\sqrt{2}}$ (3) $\tan\theta = -\sqrt{3}$

solution

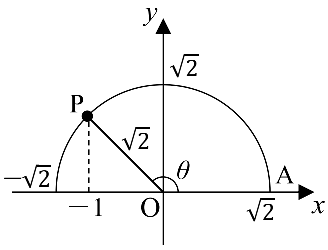
(1) When $\sin\theta = \frac{\sqrt{3}}{2}$, if points P and Q are placed on the semicircle of radius 2 as shown in the figure on the right, the required θ are $\angle$AOP and $\angle$AOQ.

Therefore, $\boldsymbol{\theta = 60°, \ 120°}$.



(2) When $\cos\theta = -\frac{1}{\sqrt{2}}$, if point P is placed on the semicircle of radius $\sqrt{2}$ as shown in the figure on the right, the required θ is $\angle$AOP.

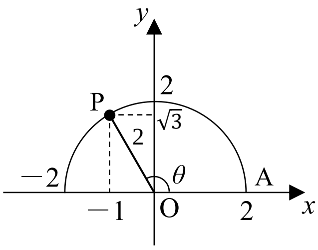
Therefore, $\boldsymbol{\theta = 135°}$.



(3) Since $\tan\theta = -\sqrt{3} = \frac{\sqrt{3}}{-1}$ and $(-1)^2 + \left(\sqrt{3}\right)^2 = 2^2$,

if we take point P on the semicircle of radius 2 as shown in the figure on the right, the required θ is $\angle$AOP.

Therefore, $\boldsymbol{\theta = 120°}$



9

$0° \leqq \theta \leqq 180°$.

(1) Find the values of $\cos\theta$ and $\tan\theta$ when $\sin\theta = \frac{15}{17}$.

(2) Find the values of $\sin\theta$ and $\cos\theta$ when $\tan\theta = -\frac{2}{11}$.

solution

(1) From $\sin^2\theta + \cos^2\theta = 1$, $\cos^2\theta = 1 - \sin^2\theta$. Substituting $\sin\theta = \frac{15}{17}$ into this gives

$$\cos^2\theta = 1 - \left(\frac{15}{17}\right)^2 = 1 - \frac{225}{289} = \frac{64}{289}.$$

(i) $\cos\theta > 0$,

$$\boldsymbol{\cos\theta} = \sqrt{\frac{64}{289}} = \boldsymbol{\frac{8}{17}} . \quad \text{Also ,} \quad \boldsymbol{\tan\theta} = \frac{\sin\theta}{\cos\theta} = \frac{15}{17} \div \frac{8}{17} = \frac{15}{17} \times \frac{17}{8} = \boldsymbol{\frac{15}{8}} .$$

(ii) $\cos\theta < 0$,

then $\boldsymbol{\cos\theta} = -\sqrt{\frac{64}{289}} = -\boldsymbol{\frac{8}{17}}$. Also , $\boldsymbol{\tan\theta} = \frac{\sin\theta}{\cos\theta} = \frac{15}{17} \div \left(-\frac{8}{17}\right) = \frac{15}{17} \times \left(-\frac{17}{8}\right) = -\boldsymbol{\frac{15}{8}}$.

(2) Substituting $\tan\theta = -\frac{2}{11}$ for $1 + \tan^2\theta = \frac{1}{\cos^2\theta}$,

we have $\frac{1}{\cos^2\theta} = 1 + \left(-\frac{2}{11}\right)^2 = 1 + \frac{4}{121} = \frac{125}{121}$.

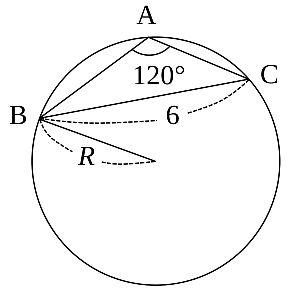
Therefore , $\cos^2\theta = \frac{121}{125}$. $0° \leqq \theta \leqq 180°$, $\tan\theta = -\frac{2}{11} < 0$, so $90° < \theta < 180°$.

From this , it is $\cos\theta < 0$. Thus , $\boldsymbol{\cos\theta} = -\sqrt{\frac{121}{125}} = -\boldsymbol{\frac{11}{5\sqrt{5}}}$.

Also , $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to $\boldsymbol{\sin\theta} = \tan\theta \cdot \cos\theta = \left(-\frac{2}{11}\right) \cdot \left(-\frac{11}{5\sqrt{5}}\right) = \boldsymbol{\frac{2}{5\sqrt{5}}}$.

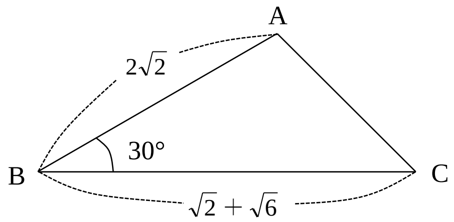
１０

In △ABC, let the lengths of sides BC, CA, and AB be denoted by a, b, and c, respectively, and the sizes of ∠A, ∠B, and ∠C be denoted by A, B, and C, respectively.



(1) Find the radius R of the circumscribed circle when $A = 120°$ and $a = 6$.

(2) Find A, b, and C when $a=\sqrt{2}+\sqrt{6}$, $B = 30°$, and $c=2\sqrt{2}$, respectively.



solution

(1) By the sine theorem, since $\dfrac{6}{\sin 120°} = 2R$, then $\dfrac{6}{\frac{\sqrt{3}}{2}} = 2R$.

Therefore, it is $\boldsymbol{R} = \left(6 \div \dfrac{\sqrt{3}}{2}\right) \times \dfrac{1}{2} = \dfrac{6}{\sqrt{3}} = \mathbf{2\sqrt{3}}$.

(2) By the cosine theorem, we have

$$b^2 = (\sqrt{2}+\sqrt{6})^2 + (2\sqrt{2})^2 - 2\cdot(\sqrt{2}+\sqrt{6})\cdot 2\sqrt{2}\cdot\cos 30° = 2 + 2\sqrt{12} + 6 + 8 - 4\sqrt{2}\,(\sqrt{2}+\sqrt{6})\cdot\frac{\sqrt{3}}{2}$$

$$= 16 + 4\sqrt{3} - 4\sqrt{3} - 12 = 4 .$$

Since $b>0$, $b=2$.

By the sine theorem, since $\dfrac{2}{\sin 30°} = \dfrac{2\sqrt{2}}{\sin C}$, $\dfrac{2}{\frac{1}{2}} = \dfrac{2\sqrt{2}}{\sin C}$.

Therefore, $\sin C = 2\sqrt{2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{\sqrt{2}}{2}$.

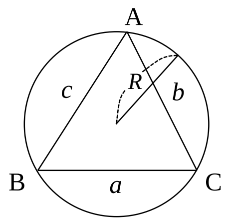
From $2\sqrt{2} < \sqrt{2}+\sqrt{6}$, since it is $C<A$, it is $C=45°$, $A=180°-30°-45°=105°$.

Thus, $\boldsymbol{(A,\ b,\ C)=(105°,\ 2,\ 45°)}$.

1 1

If $\cos A \sin C = \sin B$, what shape of triangle is △ABC ?

solution



Substituting $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, $\sin C = \dfrac{c}{2R}$, and $\sin B = \dfrac{b}{2R}$

into the given equation, respectively, yields

$$\frac{b^2 + c^2 - a^2}{2bc} \cdot \frac{c}{2R} = \frac{b}{2R}.$$

Multiplying both sides by $4bR$ yields $b^2 + c^2 - a^2 = 2b^2$.

From this, we get $a^2 + b^2 = c^2$.

Therefore, △ABC is an **isosceles triangle with ∠C＝90°** .

12

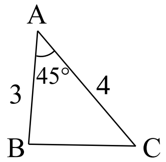
Find the area of the following △ABC .

(1) AB＝3, AC＝4, A＝45°

(2) AB＝3, AC＝5, BC＝7

solution

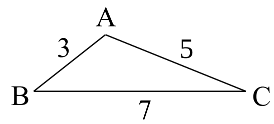
(1) $S=\frac{1}{2}\cdot 4\cdot 3\cdot \sin 45^\circ=\frac{1}{2}\cdot 4\cdot 3\cdot \frac{1}{\sqrt{2}}=\mathbf{3\sqrt{2}}$.



(2) By the cosine theorem, we have $\cos A=\frac{5^2+3^2-7^2}{2\cdot 5\cdot 3}=\frac{25+9-49}{30}=-\frac{1}{2}$.

$\sin^2 A+\cos^2 A=1$ and $0^\circ<A<180^\circ$, then $\sin A>0$, so

$$\sin A=\sqrt{1-\left(-\frac{1}{2}\right)^2}=\frac{\sqrt{3}}{2}.$$



Therefore, $S=\frac{1}{2}bc\sin A=\frac{1}{2}\cdot 5\cdot 3\cdot \frac{\sqrt{3}}{2}=\mathbf{\frac{15\sqrt{3}}{4}}$.

Alternative solution

Use Heron's formula.

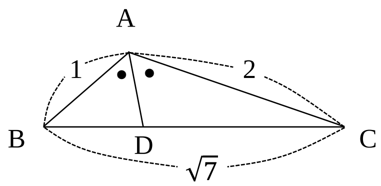
$s=\frac{a+b+c}{2}=\frac{7+5+3}{2}=\frac{15}{2}$, so it is

$$S=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{\frac{15}{2}\left(\frac{15}{2}-7\right)\left(\frac{15}{2}-5\right)\left(\frac{15}{2}-3\right)}$$

$$=\sqrt{\frac{15}{2}\cdot\frac{1}{2}\cdot\frac{5}{2}\cdot\frac{9}{2}}=\frac{15\sqrt{3}}{4}.$$

13 Fill in the blanks below.

In △ABC, when $a=\sqrt{7}$, $b=2$ and, $c=1$, $\cos A=$ (a) ,
i.e., ∠A= (b) . Therefore, the area of △ABC is
(c) . Furthermore, if the intersection of the bisector of angle A
and BC is D, then the length of AD is (d) .



solution

$\cos A = \dfrac{b^2+c^2-a^2}{2bc} = \dfrac{4+1-7}{2\cdot 2\cdot 1} = -\dfrac{\mathbf{1}}{\mathbf{2}}$. $0° \leqq \angle A \leqq 180°$, so it is $\angle A = \mathbf{120°}$.

$\triangle \mathrm{ABC} = \dfrac{1}{2}\cdot 2\cdot 1\cdot \sin 120° = \dfrac{1}{2}\cdot 2\cdot 1\cdot \dfrac{\sqrt{3}}{2} = \dfrac{\mathbf{\sqrt{3}}}{\mathbf{2}}$.

△ABD＋△ACD＝△ABC, so by the area formula, it is

$$\frac{1}{2}\cdot \mathrm{AD}\cdot 1\cdot \sin 60° + \frac{1}{2}\cdot \mathrm{AD}\cdot 2\cdot \sin 60° = \frac{\sqrt{3}}{2}.$$

From this it follows that $\dfrac{\sqrt{3}}{4}\mathrm{AD} + \dfrac{\sqrt{3}}{2}\mathrm{AD} = \dfrac{\sqrt{3}}{2}$, $\dfrac{3\sqrt{3}}{4}\mathrm{AD} = \dfrac{\sqrt{3}}{2}$.

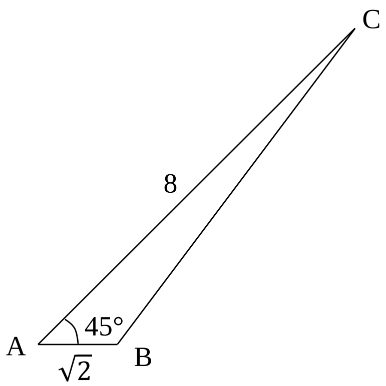
Therefore, $\mathrm{AD} = \dfrac{\sqrt{3}}{2}\cdot \dfrac{4}{3\sqrt{3}} = \dfrac{\mathbf{2}}{\mathbf{3}}$.

(a) $-\dfrac{\mathbf{1}}{\mathbf{2}}$, (b) $\mathbf{120}°$, (c) $\dfrac{\mathbf{\sqrt{3}}}{\mathbf{2}}$, (d) $\dfrac{\mathbf{2}}{\mathbf{3}}$

14

Find the radius r of the inscribed circle in $\triangle$ABC when $A=45°$, $b=8$, and $c=\sqrt{2}$.

solution



Let S be the area of $\triangle$ABC,

$$S=\frac{1}{2}bc\sin A=\frac{1}{2}\cdot 8\cdot\sqrt{2}\cdot\sin 45°$$

$$=\frac{1}{2}\cdot 8\cdot\sqrt{2}\cdot\frac{1}{\sqrt{2}}=4\,.$$

Also, $a^2=b^2+c^2-2bc\cos A=8^2+(\sqrt{2})^2-2\cdot 8\cdot\sqrt{2}\cdot\cos 45°$

$$=64+2-2\cdot 8\cdot\sqrt{2}\cdot\frac{1}{\sqrt{2}}=50\,.$$

$a>0$, so it is $a=5\sqrt{2}$.

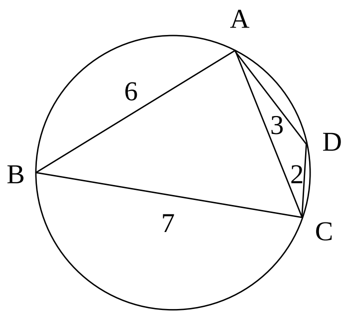
Substituting the respective values into $S=\frac{1}{2}r(a+b+c)$ yields $4=\frac{1}{2}r\left(5\sqrt{2}+8+\sqrt{2}\right)$.

$4=\left(4+3\sqrt{2}\right)r$, so it is $\boldsymbol{r}=\frac{4}{4+3\sqrt{2}}=\frac{4(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})}=\boldsymbol{6\sqrt{2}-8}$.

Study 1

In quadrilateral ABCD inscribed in a circle, find the length of diagonal AC and the area S of quadrilateral ABCD when AB＝6, BC＝7, CD＝2, and DA＝3, respectively.

solution



In △ABC, by the cosine theorem,

$AC^2=6^2+7^2-2\cdot6\cdot7\cdot\cos\angle ABC$

$=85-84\cos\angle ABC$ ……① .

In △ADC, by the cosine theorem,

$AC^2=2^2+3^2-2\cdot2\cdot3\cdot\cos\angle ADC$

$=13-12\cos(180°-\angle ABC)$

$=13+12\cos\angle ABC$ ……② .

From ① and ②, we have $85-84\cos\angle ABC=13+12\cos\angle ABC$.

Solving for this, we get $\cos\angle ABC=\frac{3}{4}$. Substituting for ①, we have $AC^2=85-84\cdot\frac{3}{4}=22$.

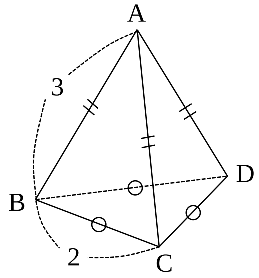
Since $AC>0$, $\mathbf{AC=\sqrt{22}}$.

Also, from $\sin\angle ABC=\sqrt{1-\left(\frac{3}{4}\right)^2}=\frac{\sqrt{7}}{4}$ and $\sin\angle ADC=\sin(180°-\angle ABC)=\sin\angle ABC$, we get

$$\boldsymbol{S}=\triangle ABC+\triangle ADC=\frac{1}{2}\cdot6\cdot7\cdot\frac{\sqrt{7}}{4}+\frac{1}{2}\cdot2\cdot3\cdot\frac{\sqrt{7}}{4}=\mathbf{6\sqrt{7}}.$$

Study 2

Find the volume of a regular triangular pyramid ABCD, as shown in the figure on the right.



solution

Draw a perpendicular line AH from vertex A to the bottom △BCD, which is

△ABH≡△ACH≡△ADH .

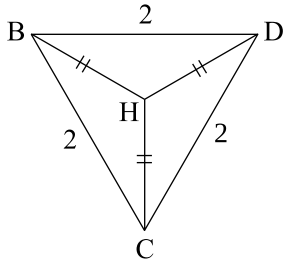
∠AHB＝∠AHC＝∠AHD ＝90° ,
AB＝AC＝AD,
AH is common.

From this, since BH＝CH＝DH , point H is the outer center of △BCD .

Therefore, BH is the radius of the circumscribed circle of △BCD, which is

$\frac{2}{\sin 60°} = 2\mathrm{BH}$. From this, we have $\mathrm{BH} = \frac{2\sqrt{3}}{3}$.

Since △ABH is a right triangle, it is



$$\mathrm{AH} = \sqrt{\mathrm{AB}^2 - \mathrm{BH}^2} = \sqrt{3^2 - \left(\frac{2\sqrt{3}}{3}\right)^2} = \frac{\sqrt{69}}{3}$$

by the Pythagorean theorem.

Also, $\triangle\,\mathrm{BCD} = \frac{1}{2}\cdot 2\cdot 2\cdot \sin 60° = \sqrt{3}$.

From the above, the volume of the regular triangular pyramid is $\frac{1}{3}\cdot\triangle\,\mathrm{BCD}\cdot\mathrm{AH} = \frac{1}{3}\cdot\sqrt{3}\cdot\frac{\sqrt{69}}{3} = \boldsymbol{\frac{\sqrt{23}}{3}}$.