

## *Differentiation and Integration*

1

Find the value of the following question for the function  $f(x)=x^2-2$ .

- (1) Average rate of change when the value of  $x$  varies from  $-2$  to  $1$
- (2) Differential coefficient at  $x=-1$
- (3) The value of  $t$  when the slope of the tangent line at point  $A(t, f(t))$  on the curve  $y=f(x)$  is  $2$

2

Differentiate the function  $f(x)=x^2+3x$  according to the definition.

3

Differentiate the following functions.

(1)  $y = x^3 - 3x^2 - 3x - 6$

(2)  $y = (x + 2)(x - 4)^2$

4

(1) Find the differential coefficient at  $x = -3$  for the following function  $f(x)$  .

①  $f(x) = 2x^2 + 4x$

②  $f(x) = x^3 + 4x^2 + x + 2$

(2) The position  $f(t)$  m of an object moving in a straight line after  $t$  seconds is represented by  $f(t) = t^2 + 3t$  .

Find the following.

① Average speed from 1 second to 5 seconds later

② Instantaneous speed after 3 seconds

5

- (1) Find the equation of the tangent line at the point  $(1, 2)$  on the curve  $y=x^3+x^2$ .
- (2) Find the equation of the tangent line drawn from point  $(1, -1)$  to the curve  $y=x^2+2x$ .

6

(1) Find the increase or decrease of the function  $y=x^3+3x^2-9x-7$ .

(2) Examine and graph the extreme values of the following functions.

①  $y=-2x^3+x^2+8x$

②  $y=-3x^3+3x^2-x+1$

7

When the cubic function  $f(x) = -x^3 + ax^2 + bx + 1$  reaches a minimal at  $x = -\frac{1}{3}$  and a maximal at  $x = 1$ , find the values of the constants  $a$  and  $b$ .

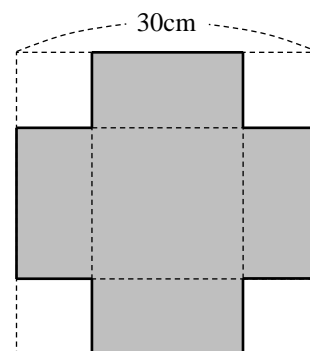
8

- (1) When the function  $f(x) = -x^3 + x^2 - 3ax + 2$  has extreme values, find the range of possible values for the constant  $a$ .
- (2) Find the range of values of the constant  $a$  such that the function  $f(x) = x^3 + 2ax^2 + 3x - 4$  has no extreme values.



9

- (1) Find the maximum and minimum values of the function  $y=x^3-2x^2$  in the interval  $-1 \leq x \leq 2$ .
- (2) Cut out squares of the same size from the four corners of a square of cardboard with 1 side of 30 cm to make a rectangular box without a lid. In this case, to maximize the volume of the box, how many centimeters should 1 side of the square to be cut out be?



10

When the cubic equation  $2x^3 - 6x + a = 0$  has 3 different real solutions, find the range of possible values of the constant  $a$ .

1 1

Prove that the inequality  $x^3 + 80 \geq 3x(x + 8)$  is satisfied when  $x \geq 0$ .

1 2

(1) Find the following indefinite integrals.

①  $\int (2x + 1) dx$

②  $\int (x^2 - 3x - 5) dx$

③  $\int (2t^2 + 1)(2t - 3) dt$

(2) Find a function  $f(x)$  satisfying  $f'(x) = 3x^2 - x$ ,  $f(2) = 7$ .

**13**

(1) Find the following definite integrals.

①  $\int_{-1}^1 (x^2 - 3) dx$

②  $\int_0^2 (2t + 1)(4t^2 - 2t + 1) dt$

③  $\int_1^3 x^2(x - 4) dx + 4 \int_1^3 x(x - 1) dx - \int_2^3 x(x + 2)(x - 2) dx$

(2) Find a function  $f(x)$  satisfying the equality  $f(x) = 2x^2 + 2x - \int_{-3}^0 f(t) dt$ .

14

Find the function  $f(x)$  satisfying the equality  $\int_a^x f(t) dt = 3x^2 + 4x + 1$   
and the value of the constant  $a$ , respectively.

15

Find the area  $S$  of the figures bounded by the following curves and lines.

(1)  $y=2x^2+2x$ ,  $x$ -axis,  $x=1$ ,  $x=2$

(2)  $y=-x^2+4$ ,  $x$ -axis

(3)  $y=x^2-3x+2$ ,  $x$ -axis

(4)  $y=x^2+2x+3$ ,  $y=-2x$

(5)  $y=(x+1)^2$ ,  $y=-x^2+5$

16

- (1) Find the area  $S$  of the figure bounded by the curve  $y=x^3-7x+6$  and the  $x$ -axis.
- (2) ① Find the equation of the tangent line drawn from point  $(2, -3)$  to the curve  $y=x^2$ .  
② Find the area  $S$  of the figure bounded by the two tangent lines and the curve  $y=x^2$  obtained in ①.