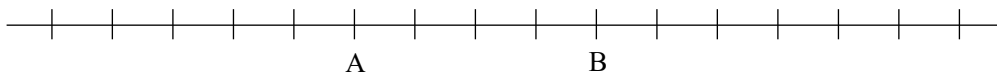


## Figure Properties

1

Figure the following points for the line segment AB.

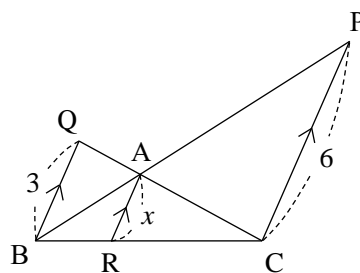
- (1) Point P that divides inside into 1 : 3
- (2) Point Q that divides outside to 1 : 2
- (3) Point R that divides outside to 7 : 3
- (4) Midpoint M



2

In the figure on the right, find the length  $x$  of the line segment.

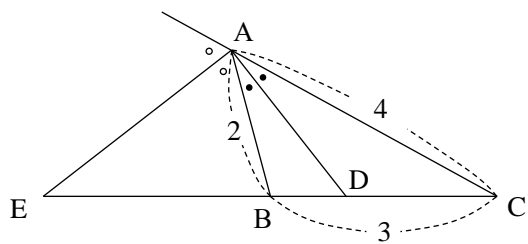
However,  $AR \parallel BQ$ ,  $AR \parallel CP$ ,  $BQ = 3$  and  $CP = 6$ .



3

In  $\triangle ABC$  where  $AB=2$ ,  $BC=3$ , and  $CA=4$ ,  $D$  is the intersection of the bisector of angle  $A$  with side  $BC$ , and  $E$  is the intersection of the bisector of the exterior angle of angle  $A$  with the extension of side  $BC$ .

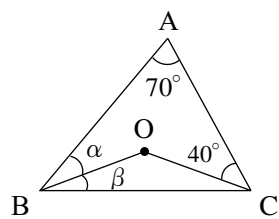
Find the length of line segment  $DE$ .



4

In the figure on the right, point O is the outer center of  $\triangle ABC$ .

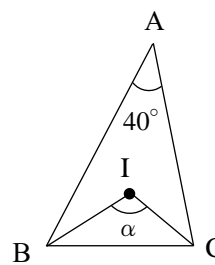
Find  $\alpha$ ,  $\beta$ .



5

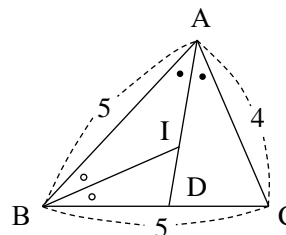
(1) In the figure on the right, point I is the interior center of  $\triangle ABC$ .

Find  $\alpha$ .



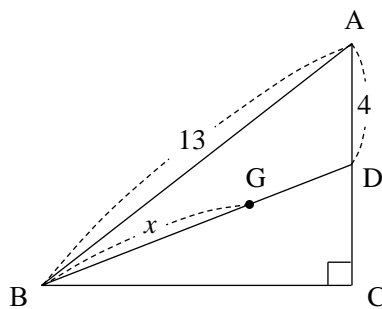
(2) In  $\triangle ABC$  where  $AB=5$ ,  $BC=5$ , and  $CA=4$ ,

let I be the interior center and D be the intersection of line AI and side BC, then find  $AI : ID$ .



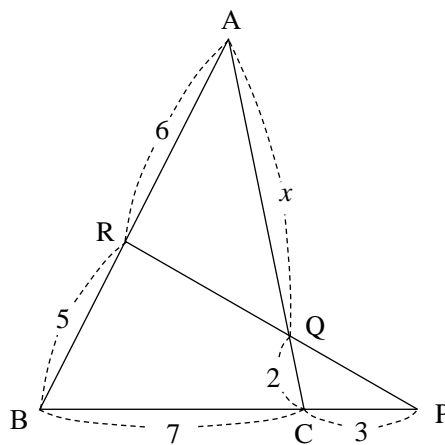
6

In the figure on the right,  
 point G is the center of gravity of  $\triangle ABC$  and  $\angle C=90^\circ$ .  
 Find the length  $x$  of the line segment.



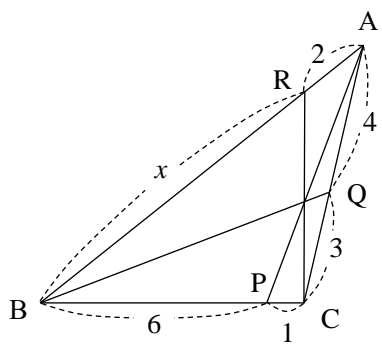
7

In the figure on the right, find the length  $x$  of the line segment.



8

In the figure on the right, find the length  $x$  of the line segment.

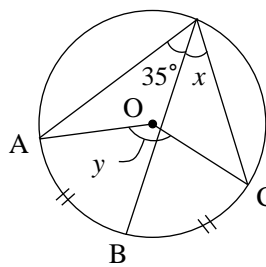




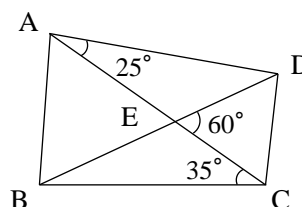
9

(1) In the figure on the right, find  $x$ ,  $y$ .

However, point O is the center of the circle and  $\widehat{AB} = \widehat{BC}$ .

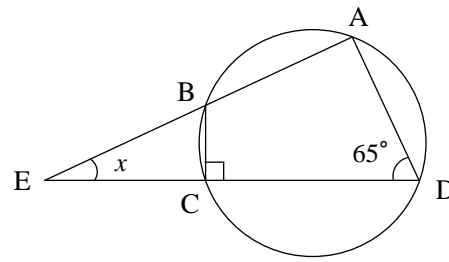


(2) In the figure on the right, are the 4 points A, B, C, and D on the same circumference?



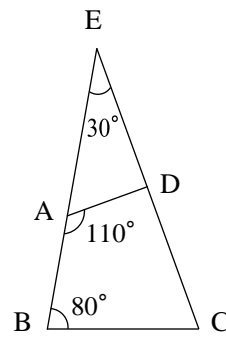
10

In the figure on the right, find  $x$  .



1 1

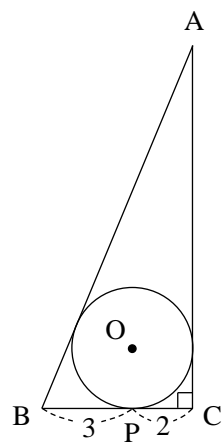
Is the right quadrilateral ABCD inscribed in a circle?



1 2

In the figure on the right, circle O is the inscribed circle of right triangle ABC with angle  $C = 90^\circ$  and point P is the contact point between side BC and circle O.

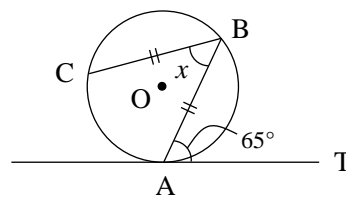
Find the lengths of sides AB and AC when  $BP = 3$  and  $CP = 2$ .



13

In the figure on the right, the line AT is tangent to the circle O at the point A.

Find  $x$ .

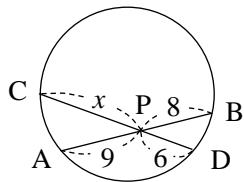


14

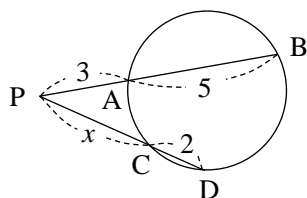
Find the value of  $x$  in the following figure.

Note that the line  $PT$  in (3) is tangent to the circle whose tangent point is  $T$ .

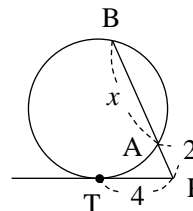
(1)



(2)

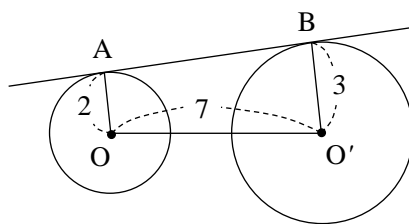


(3)



15

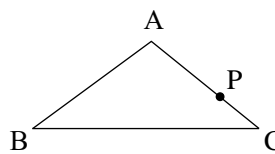
In the figure on the right, straight line AB is the common tangent of the 2 circles O and O', and points A and B are the points of contact. Find the length of line segment AB.



16

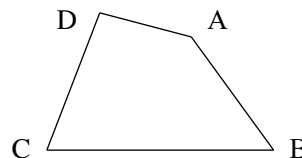
(1)  $\triangle ABC$  and a point P on side AC are given.

Draw a line passing through point P and bisecting the area of  $\triangle ABC$ .



(2) There is a quadrilateral ABCD as shown in the figure on the right.

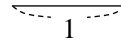
Draw a line passing through vertex A and bisecting the area of quadrilateral ABCD.





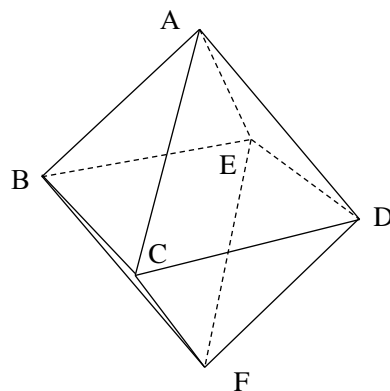
17

Given a line segment of length 1, construct a line segment of length  $\sqrt{3}$ .



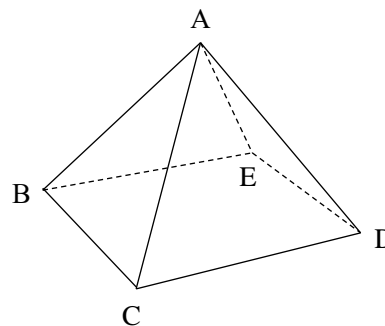
18

Find the angle between 2 lines AB and EF  
in the regular octahedron ABCDEF whose length of one side is 1,  
as shown in the figure on the right.



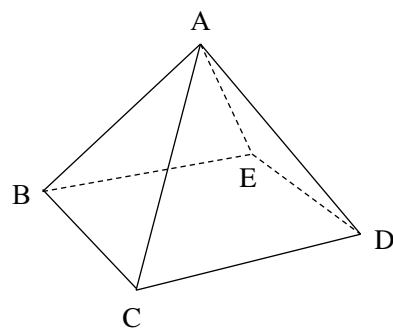
19

In a regular quadrilateral pyramid A-BCDE with side length 1 as shown in the figure on the right, find the value of  $\cos\theta$  when the angle between line AB and plane BCDE is  $\theta$ .



20

In a regular quadrilateral pyramid A-BCDE with length 1 on one side, find the value of  $\cos\theta$  if the angle between the planes ABC and BCDE is  $\theta$ .

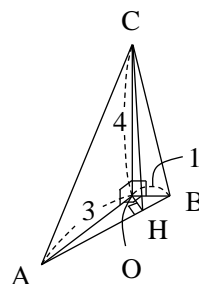


21

There are line segments  $OA$ ,  $OB$ , and  $OC$  perpendicular to each other,  
 $OA = 3$ ,  $OB = 1$ , and  $OC = 4$ .

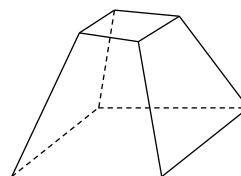
When drawing a perpendicular line  $OH$  from point  $O$  to line segment  $AB$ ,  
answer the following questions.

- (1) Find the length of the line segment  $OH$ .
- (2) Find the length of the line segment  $CH$ .
- (3) Find the area of  $\triangle ABC$ .



2 2

Verify that Euler's polyhedron theorem holds for the polyhedron shown in the figure on the right.



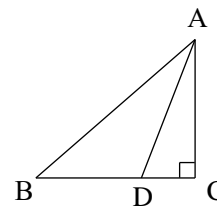
**Study**

Answer the following questions.

(1) Right triangle ABC with angle  $C = 90^\circ$ ,

let D be a point on side BC, prove that

$$AB > AD .$$



(2) Find if there exists a triangle whose 3 sides have the following lengths.

① 3, 5, 7

② 1, 2, 3