

## *Statistical Inference*

Hereafter, all root events are assumed to be equally certain.

1

When 3 balls are taken out of a bag containing 6 red balls and 3 white balls, find the probability distribution of the number  $X$  of red balls taken out. Also, find the probability  $P(X \leq 2)$ .

2

There are 30 lots with prizes as shown in the table on the right.

Find the average of the prize money when 1 of these lots is drawn.

	prize	number of lots
1st class	10000 yen	1 lot
2nd class	1000 yen	4 lots
3rd class	100 yen	25 lots

3

When 3 balls are removed from a bag containing 6 red balls and 3 white balls, find the mean, variance, and standard deviation of the number  $X$  of red balls removed.

4

Let  $X$  be the number of times the front shows up when 1 coin is tossed 4 times. Find the mean, variance and standard deviation of the random variable  $X$ .

Also, find the mean, variance and standard deviation of the random variable  $Y$  defined by  $Y=3X+2$ .

5

Let  $X$  be the number of times that a die is thrown 4 times repeatedly, and the number of times that a die with a roll of 2 or less is thrown. Find the probability distribution of  $X$ .

Also, find the probability that a die with 2 or less will be thrown 3 or more times.

6

Find the mean, variance, and standard deviation of the number of times  $X$  that a 5 appears when 1 die is thrown 360 times.

7

If the range of possible values of  $x$  for the random variable  $X$  is  $0 \leq x \leq 2$  and

its probability density function is expressed as  $f(x) = \frac{1}{2}x$  ( $0 \leq x \leq 2$ ),

find the probability  $P\left(\frac{3}{2} \leq X \leq 2\right)$ .

8

Answer the following questions.

(1) If the random variable  $Z$  follows a standard normal distribution  $N(0, 1)$ ,

find the probability that

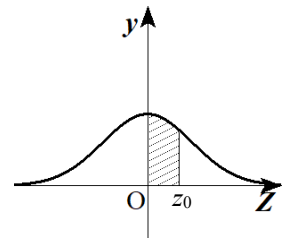
①  $P(-1 \leq Z \leq 1)$

②  $P(Z \geq -0.5)$

(2) Find  $P(2 \leq X \leq 9)$  when the random variable  $X$  follows a normal distribution  $N(1, 4^2)$ .

(3) When 100 white-fronted butterflies were collected, the mean body length was 19.6 mm, and the standard deviation was 0.5 mm.

If the body lengths of the butterflies follow a normal distribution, how many of them are more than 20 mm?



normal distribution table

$z_0$	0	~	3	~	5	6	7	8
0.2	0.0793		0.0910		0.0987	0.1026	0.1064	0.1103
~								
0.5	0.1915		0.2109		0.2088	0.2123	0.2157	0.2190
~								
0.8	0.2881		0.2967		0.3023	0.3051	0.3078	0.3106
~								
1.0	0.3413		0.3485		0.3531	0.3554	0.3577	0.3599
~								
1.2	0.3849		0.3907		0.3944	0.3962	0.3980	0.3997
~								
1.6	0.4452		0.4484		0.4505	0.4515	0.4525	0.4535
~								
1.9	0.4713		0.4732		0.4744	0.4750	0.4756	0.4761
2.0	0.4772		0.4788		0.4798	0.4803	0.4808	0.4812
~								
2.3	0.4893		0.4901		0.4906	0.4909	0.4911	0.4913
~								
2.5	0.4938		0.4943		0.4946	0.4948	0.4949	0.4951

The table above is an excerpt of only those portions relevant to this section.



9

There is a person who has a 96% success rate in Kendama.

Find the probability that the number of successes is less than 140 when this person plays Kendama 150 times.

10

Answer the following questions.

- (1) Let  $X_1$ ,  $X_2$ , and  $X_3$  be the rolls of a single die thrown 3 times, and let  $\bar{X} = \frac{X_1 + X_2 + X_3}{3}$  be the average of the rolls. Find the mean  $E(\bar{X})$  and standard deviation  $\sigma(\bar{X})$  of  $\bar{X}$ .
- (2) It is known that sweet potatoes from a certain field follow a normal distribution with mean 270g and standard deviation 30g.  
Find the probability that the sample mean  $\bar{X}$  is less than or equal to 264g when 36 randomly selected.

1 1

Answer the following questions.

- (1) It is known that the body lengths of ayu migrating up a certain river follow a normal distribution with a population standard deviation of 2.0cm, although the mean body length differs from year to year. In one year, we examined the length of 16 ayu in this river and found that the sample mean was 8.0cm. Estimate the mean length  $m$  of ayu at 95% confidence level.
- (2) We randomly selected 100 people who had won a prize in a crane game and asked them how much they spent. The sample mean was 1000 yen and the sample standard deviation was 500 yen. Estimate the average cost of taking a prize in the crane game at the 95% confidence level.

1 2

When a high school basketball player's free-throw record was randomly sampled for 144 free throws, he was successful 92 times.

Estimate the free-throw success rate of the basketball player at a 95% confidence level for the interval.

13

A 10-item nationwide disaster prevention questionnaire was conducted, in which respondents were asked whether they had prepared emergency rations and other items on a scale of  $\bigcirc$  or  $\times$ . The average number of  $\bigcirc$  was 7.00 and the standard deviation was 1.00. When this questionnaire was administered to 400 randomly selected households in one city, the average number of  $\bigcirc$  was 6.88. In this case, answer the following questions.

- (1) Can the city's results be considered comparable to the national level? Test at 5% level of significance. Also, test at the 1% level of significance.
- (2) The mayor of one city has a stronger interest in whether these results are less than the national ones. Can we conclude that a city's result is less than the national result? Test at 1% level of significance.