Probability

1

(1) When 2 coins are tossed at the same time, find the probability that 1 coin comes out face up.

(2) When 2 dice are thrown at the same time, find the probability that the product of the rolls of the dice is 12.

solution

(1) The number of all possible cases is

$2^2=4$ (ways).

Of these, there are 2 ways in which 1 front can appear: (front, back) or (back, front).

Therefore, the probability of seeking is

$\frac{2}{4}=\mathbf{\frac{1}{2}}$.

The 2 coins are considered separately. The total events are

(front, front), (front, back), (back, front), and (back, back),

and these root events are equally certain.

If we consider the 2 coins without distinguishing between them, and assume that there are 3 total events,

(front, front), (front, back), and (back, back),

we cannot calculate the probabilities because (front, front) and (front, back) are equally improbable.

(2) The total number of ways the dice can come out is $6\times6=36$ (ways).

There are 4 ways that the product of the dice is 12: (2, 6), (3, 4), (4, 3), and (6, 2).

Therefore, the probability of seeking is $\frac{4}{36}=\mathbf{\frac{1}{9}}$.

2

When 3 boys and 3 girls line up in a row, find the probability that the boys and girls alternate.

solution

There are 6! ways to line up 6 people in a row.

Of these, boys and girls may alternate in 2 cases:

boy-girl-boy-girl-boy-girl and girl-boy-girl-boy-girl-boy.

In each case, there are 3! ways for the boys and 3! ways for the girls, so there are 2 × 3! × 3! ways for each of them.

Therefore, the probability of seeking is $\frac{2 \times 3! \times 3!}{6!} = \frac{2 \times 3 \cdot 2 \cdot 1 \times 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \mathbf{\frac{1}{10}}$.

3

Find the probability of drawing exactly 1 winning lottery ticket when 3 lots are drawn at the same time from 10 lots containing 3 winning lots.

solution

Considering that all lots are distinguishable, the number of cases in which 3 lots are drawn out of 10 lots is ${}_{10}C_3$.

Of these, the number of cases where 1 from 3 hits and 2 from 7 misses are subtracted is ${}_3C_1 \times {}_7C_2$（ways）.

Therefore, the probability of seeking is $\dfrac{{}_3C_1 \times {}_7C_2}{{}_{10}C_3} = \dfrac{3 \times \dfrac{7 \cdot 6}{2}}{\dfrac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}} = \mathbf{\dfrac{21}{40}}$.

4

When 3 people play rock-paper-scissors once, find the probability of a game of rock-paper-scissors.

solution

There are a total of $3^3=27$ different ways to make a move in a three-player game of rock-paper-scissors.

A game of rock-paper-scissors is played when all 3 players have the same move and when all 3 players have different moves, and these events are mutually exclusive.

(i) Probability that all 3 players will play the same rock-paper-scissors move

When all players make “rock”, “ scissors”, or “paper”, it is $\frac{3}{27}$.

(ii) Probability that all 3 players will make different rock-paper-scissors moves

The number of cases where 3 players make 3 different rock-paper-scissors moves (rock, scissors, paper) is $3!=6$ (ways), which is $\frac{6}{27}$.

Therefore, the probability of a game of rock-paper-scissors is $\frac{3}{27}+\frac{6}{27}=\frac{9}{27}=\mathbf{\frac{1}{3}}$.

5

When 1 card is drawn from 50 cards numbered from 1 to 50, find the probability that the number is divisible by 3 or 7.

solution

The number of cases in which event A, which draws a number divisible by 3 from 50 cards, occurs is 16 from $A = \{3 \cdot 1,\ 3 \cdot 2,\ \cdots\cdots,\ 3 \cdot 16\}$.

The number of cases in which event B, which draws a number divisible by 7 from 50 cards, occurs is 7 from $B = \{7 \cdot 1,\ 7 \cdot 2,\ \cdots\cdots,\ 7 \cdot 7\}$.

Also, since the product event $A \cap B$ is the event of subtracting a number divisible by 21, there are 2 ways from $A \cap B = \{21 \cdot 1,\ 21 \cdot 2\}$.

Therefore, the probability of seeking is $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{16}{50} + \frac{7}{50} - \frac{2}{50} = \mathbf{\frac{21}{50}}$.

6

(1) A 12-piece product contains 3 defective products. Find the probability that 2 of these products contain defective products when they are taken out.

(2) Find the following probabilities when 3 dice are thrown at the same time.

① Probability that the maximum number of dice rolls that come out is less than or equal to 4

② Probability that the maximum number of dice rolls that will appear is 4

solution

(1) The event that a defective product is included is event $\overline{A}$, which is an after-event of event A in which neither of the 2 products is defective.

Since it is $P(A)=\dfrac{{}_9\mathrm{C}_2}{{}_{12}\mathrm{C}_2}=\dfrac{\frac{9\cdot 8}{2\cdot 1}}{\frac{12\cdot 11}{2\cdot 1}}=\dfrac{6}{11}$, the probability we seek is $P(\overline{A})=1-P(A)=1-\dfrac{6}{11}=\mathbf{\dfrac{5}{11}}$.

(2) ① When 3 dice are thrown simultaneously, the number of all possible cases is 6^3 (ways).

The maximum number of dice that can be rolled is less than or equal to 4 if all 3 dice rolls are less than or equal to 4. There are 6^3 (ways) such cases.

Therefore, the probability of seeking is $\dfrac{4^3}{6^3}=\mathbf{\dfrac{8}{27}}$.

② There are 3^3 ways in which all the 3 dice can be less than or equal to 3.

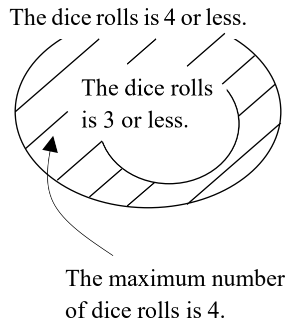
The number of cases where the maximum number of dice rolls is 4 is

(the number of cases where the result is 4 or less)

− (the number of cases where the result is 3 or less).

Therefore, there are 4^3-3^3 ways.

Thus, the probability of seeking is $\dfrac{4^3-3^3}{6^3}=\mathbf{\dfrac{37}{216}}$.



7

Pouch A contains 3 red balls and 5 white balls, and pouch B contains 4 red balls and 2 white balls. When 1 ball is removed from pouch A and 1 ball from pouch B, find the probability that the 2 balls are of different colors.

solution

The attempt to remove balls from pouch A and the attempt to remove balls from pouch B are independent.

The 2 balls become different colors when a red ball is removed from pouch A and a white ball from pouch B, and when a white ball is removed from pouch A and a red ball from pouch B. These events are mutually exclusive.

(i) Probability of removing a red ball from pouch A and a white ball from pouch B

The probability of retrieving 1 red ball from pouch A is $\frac{3}{8}$

and the probability of retrieving 1 white ball from pouch B is $\frac{2}{6}=\frac{1}{3}$.

Therefore, the probability of taking a red ball from pouch A and a white ball from pouch B is $\frac{3}{8}\times\frac{1}{3}=\frac{3}{24}$.

(ii) Probability of removing a white ball from pouch A and a red ball from pouch B

The probability of retrieving 1 white ball from pouch A is $\frac{5}{8}$

and the probability of retrieving 1 red ball from pouch B is $\frac{4}{6}=\frac{2}{3}$.

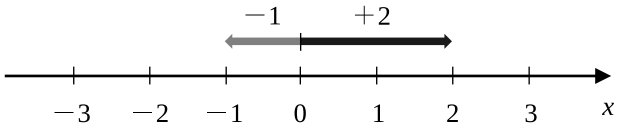
Therefore, the probability of taking a white ball from pouch A and a red ball from pouch B is $\frac{5}{8}\times\frac{2}{3}=\frac{10}{24}$.

Thus, the probability that the 2 balls will be of different colors is $\frac{3}{24}+\frac{10}{24}=\mathbf{\frac{13}{24}}$.

8

(1) Point P is on the x-axis.

When a dice is thrown and a multiple of 3 is obtained, P moves forward on the x-axis by 2 in the positive direction, and when a dice is not a multiple of 3, P moves forward on the x-axis by 1 in the negative direction.



Find the probability that P starting from the origin is at the point $x=1$ when the dice are thrown 5 times.

(2) 2 teams, A and B, will play a volleyball match. When the winner is the one who wins the first 3 sets, find the following probabilities. However, assume that the probability of A beating B in 1 set of games is $\frac{2}{3}$ and the probability of B beating A is $\frac{1}{3}$.

① Probability of A winning in the 3rd set

② Probability of A winning in the 4th set

③ Probability of A winning

(3) When tossing 6 coins, find the probability that there will be 3 fronts and 3 backs.

solution

(1) Let r be the number of times a dice is thrown 5 times and the number of times it comes up with a multiple of 3, the number of times it comes up with a dice that is not a multiple of 3 is $5-r$.

In this case, the x-coordinate of point P is $x=2\times r+(-1)\times(5-r)=3r-5$.

Substituting $x=1$, $r=2$ from $1=3r-5$.

Therefore, we can find the probability that a dice rolls of a multiple of 3 will appear exactly twice.

Dice rolls of multiples of 3	: 3, 6
Dice rolls that are not multiples of 3	: 1, 2, 4, 5

The probability that a dice thrown once will have a multiple of 3 is $\frac{2}{6}=\frac{1}{3}$.

Thus, the probability of seeking is ${}_5C_2\left(\frac{1}{3}\right)^2\left(1-\frac{1}{3}\right)^{5-2}=\frac{5\cdot 4}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3=\mathbf{\frac{80}{243}}$.

(2) ① A wins the 3rd set when

A wins three in a row,

which is $\left(\frac{2}{3}\right)^3 = \mathbf{\frac{8}{27}}$.

② A wins in the 4th set when

A has won 2 games by the third set

and A wins in the 4th set,

which is ${}_3C_1\left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} \times \frac{2}{3} = \mathbf{\frac{8}{27}}$.

It is incorrect to assume ${}_4C_1\left(\frac{2}{3}\right)^3 \cdot \frac{1}{3}$ since A is 3-1.

This includes the case where A is win-win-win-lose, in which case the winner is decided in the 3rd set and the 4th set is invalid.

Note that the final set is won by the winning team.

③ A may win in

(i) the 3rd set (ii) the 4th set (iii) the 5th set,

which are mutually exclusive.

From ① and ②, the probability of (i) is $\frac{8}{27}$ and that of (ii) is $\frac{8}{27}$,

so we can find the probability of (iii).

A wins in the 5th set when A has won 2 games by the 4th set and A wins in the 5th set, which is

$${}_4C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 \times \frac{2}{3} = \frac{4\cdot 3}{2}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 = \frac{16}{81}.$$

Thus, the probability of seeking is $\frac{8}{27} + \frac{8}{27} + \frac{16}{81} = \mathbf{\frac{64}{81}}$.

(3) The probability we seek is the same as the probability of getting 3 fronts and 3 backs when tossing a coin 6 times, and can be considered as the probability of repeated trials.

Since the probability of getting a " front " and the probability of getting a " back " are both $\frac{1}{2}$, the probability of seeking is

$${}_6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 = \frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1}\left(\frac{1}{2}\right)^6 = \mathbf{\frac{5}{16}}.$$

For example, the probability of getting 3 correct when there are 6 questions in the true or false formula and marking them with a true and a false, or the probability of splitting exactly 3 people each time when 6 people are divided into 2 groups by “rock” and “paper”, is $\frac{5}{16}$ by the same calculation.

9

There are 10 lots including 4 winners. When X draws the first lottery ticket and Y draws the next ticket without putting it back, find the probability that Y draws the winning ticket.

solution

Let A be the event in which X draws the winning lottery ticket and

B be the event in which Y draws the winning lottery ticket.

There are 2 cases in which Y draws a winning lottery ticket.

(i) If X draws a winning lottery ticket and Y also draws a winning lottery ticket.

$$P(A \cap B) = P(A)P_A(B) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$$

(ii) If X draws a losing lottery ticket and Y draws a winning lottery ticket.

$$P(\overline{A} \cap B) = P(\overline{A})\, P_{\overline{A}}(B) = \frac{6}{10} \times \frac{4}{9} = \frac{24}{90}$$

Since (i) and (ii) are mutually exclusive, the probability of seeking is

$$P(B) = P(A \cap B) + P(\overline{A} \cap B) = \frac{12}{90} + \frac{24}{90} = \frac{36}{90} = \mathbf{\frac{2}{5}}.$$

１０

At a certain supermarket, a 700-yen boxed meal is discounted at 7:00 p.m. with the probabilities shown in the table on the right.
Find the expected value of this boxed meal at 7:00 p.m.
Also, if this boxed meal is sold at 7:00 p.m. at a 20% discount, should I buy it or not?

discount rate	probability
50% off	$\frac{1}{10}$
20% off	$\frac{6}{10}$
no discount	$\frac{3}{10}$
total	1

solution

The amount sought is

$$700 \times (1-0.5) \times \frac{1}{10} + 700 \times (1-0.2) \times \frac{6}{10} + 700 \times \frac{3}{10} = \frac{350 + 3360 + 2100}{10} = \mathbf{581(yen)}\,.$$

Also, the 20% off price on this boxed meal is　$700\times(1-0.2)=560$（yen）.

This is cheaper than what is expected for a boxed meal, so **I prefer to buy it.**

Study

Two machines A and B are producing the same product. The product of machine A contains 0.3% defective products and the product of machine B contains 0.1% defective products. When 400 products from machine A and 600 products from machine B are extracted and 1 product is taken out after stirring well, find the following probabilities.

(1) Probability of defective product

(2) Probability that the defective product is the product of machine A

solution

(1) Let A be the event that the taken out 1 product is the product of machine A,

B be the product of machine B, and

E be the event that the taken product is defective.

$$P(A)=\frac{400}{1000},\ P(B)=\frac{600}{1000},\ P_A(E)=\frac{3}{1000},\ P_B(E)=\frac{1}{1000}.$$

Therefore, $P(E)=P(A\cap E)+P(B\cap E)=P(A)P_A(E)+P(B)P_B(E)$

$$=\frac{400}{1000}\times\frac{3}{1000}+\frac{600}{1000}\times\frac{1}{1000}=\frac{12+6}{10000}=\mathbf{\frac{9}{5000}}.$$

(2) The probability we seek is $P_E(A)$, so

$$P_E(A)=\frac{P(E\cap A)}{P(E)}=\frac{P(A\cap E)}{P(E)}=\frac{12}{10000}\div\frac{9}{5000}=\mathbf{\frac{2}{3}}.$$